ekf

stepan.ozana

February 2025

1 Introduction

EKF - Linear System Model

This document describes the use of the Extended Kalman Filter (EKF) in the REXYGEN environment for state estimation of a linear system.

System Description

The system is described by the following differential equations:

$$\frac{dx(t)}{dt} = f(x(t), u(t)) + w(t),$$

$$y(t) = h(x(t), u(t)) + v(t),$$

$$\hat{x}(t) \sim \mathcal{N}(x, P),$$

$$w(t) \sim \mathcal{N}(0, Q),$$

$$v(t) \sim \mathcal{N}(0, R).$$

where:

- $x(t)$ is the state vector, - $u(t)$ is the input vector, - $y(t)$ is the output vector, - $w(t)$ is the process noise, - $v(t)$ is the measurement noise, - $\hat{x}(t)$ is the estimated state vector, - x is the mean (expected value), - P is the state estimation covariance matrix, - Q is the process noise covariance matrix, - R is the measurement noise covariance matrix.

State Model of the System

The system dynamics are defined by the function $f(x(t), u(t))$:

$$f(x(t), u(t)) = \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \\ \frac{dx_4(t)}{dt} \end{bmatrix} = \begin{bmatrix} -x_1(t) + x_2(t) \\ -x_2(t) + x_3(t) \\ -x_3(t) + x_4(t) \\ -x_4(t) + u_1(t) \end{bmatrix}.$$

Jacobian of the State Function

The Jacobian $f(x,u)$ with respect to the state vector x is given as:

$$\frac{df(x(t),u(t))}{dx} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Output Measurements

The first set of output measurements (which, in this example, depend linearly on the state variables) for input EKF:nz = 1 is defined as:

$$y(t) = h_1(x(t),u(t)) = \begin{bmatrix} x_1(t) \\ 2x_2(t) + 0.5x_4(t) \end{bmatrix},$$

with the corresponding Jacobian:

$$\frac{dh_1(x(t),u(t))}{dt} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0.5 \end{bmatrix}.$$

The second set of output measurements (for input EKF:nz = 2) is not used.

The system description is programmed into the REXLANG block.